4.5 Exponential and Logarithmic Equations

❖ Exponential Equations

Methods of Solving Exponential Equations

I. Use Like Bases

If $b^M = b^N$, then $M = N$.

1.) Rewrite the equation in the form $b^M = b^N$.
2.) Set $M = N$.
3.) Solve for the variable.

Ex. Solve each exponential equation.

a.) $5^{3x-1} = 125$

b.) $4^x = 32$

c.) $7^{\frac{x-2}{6}} = \sqrt{7}$

d.) $e^{x+4} = \dfrac{1}{e^{2x}}$

II. Use Logarithms

1.) Isolate the exponential expression.

2.) Take "ln" on both sides for bases other than 10.
Take "log" on both sides for base 10.

3.) Simplify using one of the following properties:

$\ln b^x = x \ln b$ or $\ln e^x = x$ or $\log 10^x = x$.

4.) Solve for the variable.

Ex. Solve each exponential equation. Give exact solution. Then use a calculator to obtain a decimal approximation, correct to four decimal places, for the solution.

a.) $10^x = 8.07$

b.) $9e^x = 107$

c.) $e^{4x-5} - 7 = 11,243$

d.) $7^{4x-1} = 3^{5x}$

❖ Logarithmic Equations

Methods of Solving Logarithmic Equations

I. Use the Definition of a Logarithm

1.) Express the equation in the form $\log_b M = c$ where c is a constant. If necessary, combine multiple logs into a single log. (Use Sec. 4.4 Properties of Logarithms)

2.) Rewrite the equation in exponential form $b^c = M$.

3.) Solve for the variable.

4.) Always check solutions in the original log equation to make sure they do not create the log of a negative number $(M > 0)$.

Ex. Solve each logarithmic equation. Give the exact answer. Then, where necessary, use a calculator to obtain a decimal approximation, correct to four decimal places, for the solution.

a.) $3\log_2(x-1)=15$

b.) $6\ln(2x)-7=23$

c.) $\log_6(x+5)+\log_6 x=2$

d.) $\log_2(x-3)+\log_2 x-\log_2(x+2)=2$

II. Use the One-to-One Property

If $\log_b M = \log_b N$, then $M = N$.

1.) Express the equation in the form $\log_b M = \log_b N$.

2.) Set $M = N$.

3.) Solve for the variable.

4.) Always check solutions in the original log equation to make sure they do not create the log of a negative number $(M > 0 \text{ and } N > 0)$.

Ex. Solve each logarithmic equation. Give the exact answer. Then, where necessary, use a calculator to obtain a decimal approximation, correct to four decimal places, for the solution.

a.) $\log_2(3x+1) = \log_2 5x$

b.) $\log(5x+1) = \log(2x+3) + \log 2$

c.) $\log(x+7) - \log 3 = \log(7x+1)$

d.) $\log(x+3) + \log(x-2) = \log 14$